A NEW ENERGY TRANSFER MODEL FOR TURBULENT FREE SHEAR FLOW

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Abstract

A new model for the energy transfer mechanism in the large-scale turbulent kinetic energy equation is proposed. An estimate of the characteristic length scale of the energy containing large structures is obtained from the wavelength associated with the structures predicted by a weakly nonlinear analysis for turbulent free shear flows. With the inclusion of the proposed energy transfer model, the weakly nonlinear wave models for the turbulent large-scale structures are self-contained and are likely to be independent flow geometries. The model is tested against a plane mixing layer. Reasonably good agreement is achieved. Finally, it is shown by using the Liapunov function method, the balance between the production and the drainage of the kinetic energy of the turbulent large-scale structures is asymptotically stable as their amplitude saturates. The saturation of the wave amplitude provides an alternative indicator for flow self-similarity.

Introduction

Many experiments have reported the presence and the importance of large-scale coherent structures in turbulent free shear flows for different flow configurations and operating conditions. For example, Winant and Browand[1] and Brown and Roshko[2] first observed these structures in low speed free shear layers and Papamoschou and Roshko[3] in supersonic free shear layers. Morris and Giridharan[4] and Liou and Morris[5] have constructed turbulent models to simulate the turbulent large-scale structures explicitly. The models they developed are based on a weakly nonlinear theory. Briefly, the local characteristics of the large-scale turbulent structure are described by linear instability waves. Their amplitude are determined by evolution equations derived from the turbulent kinetic energy equation. The predictions in Morris and Giridharan[4] agreed very well with measured data for the growth of the compressible shear layer for a wide range of free stream operating conditions, including the effects of free stream Mach number. Two models were developed and implemented into a mean flow prediction scheme for an incompressible free shear layer in Liou and Morris[5]. The first models the averaged development of the shear layer and the second simulates a single realization of the passage of a train of large-scale structures. The model predictions have shown a reasonable agreement with measurements and demonstrated the feasibility of the more general approach for other free shear flows. This short analysis is built upon the weakly nonlinear wave models developed by Liou and Morris[5]. A model for the turbulent length scale is constructed and applied in the calculations of the energy transfer from the large structures to the small structures. The new model characterizes the energy transfer by the dynamics of the large-scale structure alone. This feature could facilitate the application of the weakly nonlinear models to free shear flows of engineering interests. These include turbulent free shear flows of complex geometries and at various operating conditions.

In Liou and Morris[5], the random flow properties are split into three components,

$$\tilde{f}_i = F_i + f_i + f_i' \tag{1}$$

The fluctuation with respect to the long time-average component, F_i, is separated into a component representing the large-scale motion, f_i, and one representing the residual fluctuations, f_i'. The long time-average of the instantaneous value is denoted by an overbar:

$$\bar{\tilde{f}}_i = F_i = \frac{1}{T_1} \int_0^{T_1} \tilde{f}_i \mathrm{dt} \tag{2}$$

The thin-layer approximations are used to reduce the governing equations for the mean flow to the following from:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \tag{3}$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + \frac{\partial}{\partial x}(\overline{u^2} - \overline{v^2}) + \frac{\partial}{\partial y}(\overline{uv}) = \frac{1}{R_e}\frac{\partial^2 U}{\partial y^2}. \tag{4}$$

For the large-scale fluctuation, a separable form of solution was assumed:

$$\{\mathbf{u},\ \mathbf{v},\ \mathbf{p}\} = A(x)\ [\hat{u}(y), \hat{v}(y), \hat{p}(y)]\ exp\ [i(\alpha x - \omega t)] \tag{5}$$

The bold face quantities denote a complex solution whose real part describe the physical properties of the large-scale structures. $\alpha\ (= \alpha_r + i\alpha_i)$ denotes a complex wavenumber and ω the frequency. The governing equations for the local distributions of the large structures can be reduced to the Rayleigh equation in terms of $\hat{v}$:

$$\left\{(\ \alpha U\ -\ \omega\ )\ (\ \frac{\mathrm{d}}{\mathrm{d}y^2}\ -\ \alpha^2\ )\ -\ \alpha\frac{\mathrm{d}^2 U}{\mathrm{d}y^2}\right\}\hat{v} = 0 \tag{6}$$

The amplitude $A(x)$ appears as a parameter in the local calculation for the $\hat{u}, \hat{v}, \hat{p}$ and is determined separately from the large scale turbulent kinetic energy equation:

$$U_j\frac{\partial k}{\partial x_j} = -\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - \frac{\partial}{\partial x_j}(\overline{u_j k} + \frac{\overline{p u_j}}{\varrho}) - \overline{(-<u_i' u_j'>)\frac{\partial u_i}{\partial x_j}}$$

$$-\frac{\partial}{\partial x_j}\overline{(u_i <u_i' u_j'>)} + \text{viscous terms} \tag{7}$$

where $k = \frac{1}{2}\overline{u_i u_j}$. k denotes the turbulent kinetic energy of the large-scale structure. $<>$ represents a short time-average with an average interval much smaller than T_1 but much larger than the characteristic time scale of the background small-scale fluctuation[7]. The interaction terms, the third term on the right hand side of equation (7), describe the transfer of large-scale energy, presumably, to the small scales where energy is eventually dissipated by viscosity. The detailed analysis of the weakly nonlinear wave models and the numerical solution procedure used here can be found in Liou and Morris[5].

The Energy Transfer Model

The spectral energy transfer results from the interactions between turbulent fluctuations of different scales. For the weakly nonlinear wave turbulence models, the energy transfer is of crucial importance in the determination of the wave amplitude and need to be considered carefully. Very little information, experimental and theoretical, are available regarding the stresses, $-<u'_i u'_j>$. Through dimensional argument, it may be assumed that the energy transferred out of the large structures depends on[8],

$$C_1 \frac{u^3}{l} \tag{8}$$

where u and l are the characteristic velocity and length scales of the energetic large eddies, respectively. In one of the models that were developed earlier, which was referred to as the Model I in Liou and Morris[5], it was assumed that,

$$u = k^{\frac{1}{2}} \tag{9}$$

and

$$l = \delta \tag{10}$$

δ stands for the width of the mean flow defined by the transverse distance between points where the streamwise mean velocity is 0.9 and 0.1. The predictions by employing the Model I agreed with measurements for the mean velocity and the shear layer growth rate. Here, we propose a new model of the energy transfer from the large scales to the small scales. It explores a unique characteristic of the weakly nonlinear models. The model expresses the spectral energy transfer by the dynamics of the large-scale structures alone, regardless of the geometries of the mean flow.

The weakly nonlinear analysis seeks normal mode solution of the large-scale turbulent fluctuation. Locally, they are described by the linearized Euler equation. On the other hand, the spatial extent of each of the mode of the large-scale structures could be regarded as being determined by the wavenumber, α_r. Therefore, the proposition here is to estimate the characteristic size of the large scales as the wavelength associated with the structure that are predicted by the weakly nonlinear analysis. That is,

$$l = l_w = \frac{2\pi}{\alpha_r} \tag{11}$$

where l_w denotes the wavelength. With the wavelength as a length scale, equation (8) becomes,

$$C_2 \frac{k^{\frac{3}{2}}}{l_w} \tag{12}$$

This is the resulting model for the energy transfer from the large scale the the small scale. This estimate is in accord with the classic assumption of turbulence theory that dissipation ".. proceeds at a rate dictated by the inviscid inertia behavior of the large eddies."[8]. Computationally, since the wavenumber is already a part of the solution of the equations for the large-scale fluctuation, this model involves no extra efforts in estimating the characteristic size of the energy containing large scales. This rather simple model provides a closure to the equations of the large-scale structure and, therefore, render the weakly nonlinear wave description of the large-scale structure self-contained. The self-contained nature of the weakly nonlinear wave turbulence models may be important in their future applications to other turbulent free shear flows.

Results and Discussion

The model is tested against an incompressible one-stream mixing layer. To make the matter simple, we choose to predict only the averaged, mean quantities of the shear layer. Note that, in addition to the mean flow prediction, Liou and Morris[5] also calculated the time-dependent evolution of the turbulent mixing layer at the large scale. Since it is the most unstable mode that interacts most strongly with the mean flow[5], for efficiency, the most amplifying local instability is used to characterize the average, overall interactions between the mean and the large scale motions. Therefore, in the present formulation, the characteristic length scale l_w is determined only by the locally most unstable modes.

Figure 1 shows the estimated length scales for the two models, i.e., the mean width model and the present wavelength model. The wavelength is about one order of magnitude larger than the width of the mean flow, δ. The model constant, C_2, therefore, can be roughly one order of magnitude higher than the C_1 used in Liou and Morris[5]. Numerical runs with different values of C_2 showed that small changes in the value of C_2 had no significant effect on the flow development. Quantitatively,, for $C_2 = 22$, the spreading rate of the mean flow, $\mathrm{d}\delta/\mathrm{d}x = 0.142$ and for $C_2 = 20$, $\mathrm{d}\delta/\mathrm{d}x = 0.158$, both of which are within the experimental scatter. In the following figures, the results are shown for $C_2 = 20$.

Figure 2 shows the predicted evolution of the streamwise mean velocity profiles with axial distance. η is a similarity coordinate,

$$\eta = \frac{y - y_{1/2}}{x - x_0} \tag{13}$$

where $y_{1/2}$ denotes the location where the local mean velocity is one half of the free stream velocity. The predicted self-similar profiles agree well with that compiled by Patel[9] except at the low speed edge of the layer. Similar differences were also observed by Liou and Morris[5]. They attributed this difference to the single mode representation of the entire large scale spectrum and the uncertainties in the measurements in this region resulting from the local large changes in the instantaneous flow direction. Figure 3 shows the calculated Reynolds stress distributions in the self-similar coordinate. The difference between the large-scale Reynolds stresses calculated by the weakly nonlinear model and the total Reynolds stress distribution measured by Patel does not necessarily mean that the small-scale should be included. A better agreement may be obtained if a broad range of instability waves are included. At the outer edge of the low speed side, the weakly nonlinear model predicted that the momentum transport by the large-scale fluctuation is counter-gradient, a phenomenon that has been observed in many experiments.[10,11]

The streamwise evolution of the amplitude of the large-scale structures is shown in figure 4. After a region of establishment, the amplitude reaches a saturated value. In this region, the rate of the production of the large-scale turbulent kinetic energy from the mean flow is balanced by the rate of energy transfer from the large scales to the small scales. Note that, for the present energy transfer model, the amplitude equation becomes,

$$\frac{\mathrm{d}A^2}{\mathrm{d}x} = G_3(x)\, A^2 - G_4(x)\, A^3 \tag{14}$$

G_3 and G_4 denote the normalized positive definite integrals of the production terms and interaction terms across the layer, respectively. The critical points of the nonlinear equation (14), where $\frac{\mathrm{d}A^2}{\mathrm{d}x} = 0$, are $A_1 = 0$ and $G_4(x_2)A_2 = G_3(x_2)$. Simple analyses by applying the Liapunov function method[12] show that A_1 is an unstable critical point. Any small disturbances to A_1, say A_1' would grow exponentially. In fact,

$$(A_1^2)' \approx e^{G_3(x_1)\, x} \tag{15}$$

A_2, on the other hand, is asymptotically stable. A disturbance about the A_2, say A_2', would decay exponentially,

$$(A_2^2)' \approx e^{-\frac{G_3(x_2)}{2}\, x} \tag{16}$$

Note that the value of G_3 in the equations (15) and (16) are taken as their values at the corresponding critical points. The saturated value of the amplitude, A_2, is an asymptotically equilibrium value. It indicates an asymptotically equilibrium state of the large-scale structures. The simple instability analyses also show that any deviation away from this equilibrium state would be damped out exponentially. Consequently, the saturation of the wave amplitude may provide an indication of the the self-similarity of the flow in terms of the development of the large-scale structures.

The model proposed here provides a physically reasonable and self-contained representation of the energy transfer from the large scale to the small scale. For the mixing layer tested here, the results seem rather encouraging. It may be argued that, with a more realistic multi-mode representation of the large scale spectrum, modifications to the value of C_2 should be minimal in the application of this model to free shear flows of other more complex geometries. A calculation of the axisymmetric jets represents the best further test of the model. Efforts to perform this non-trivial calculation is underway and will be reported later.

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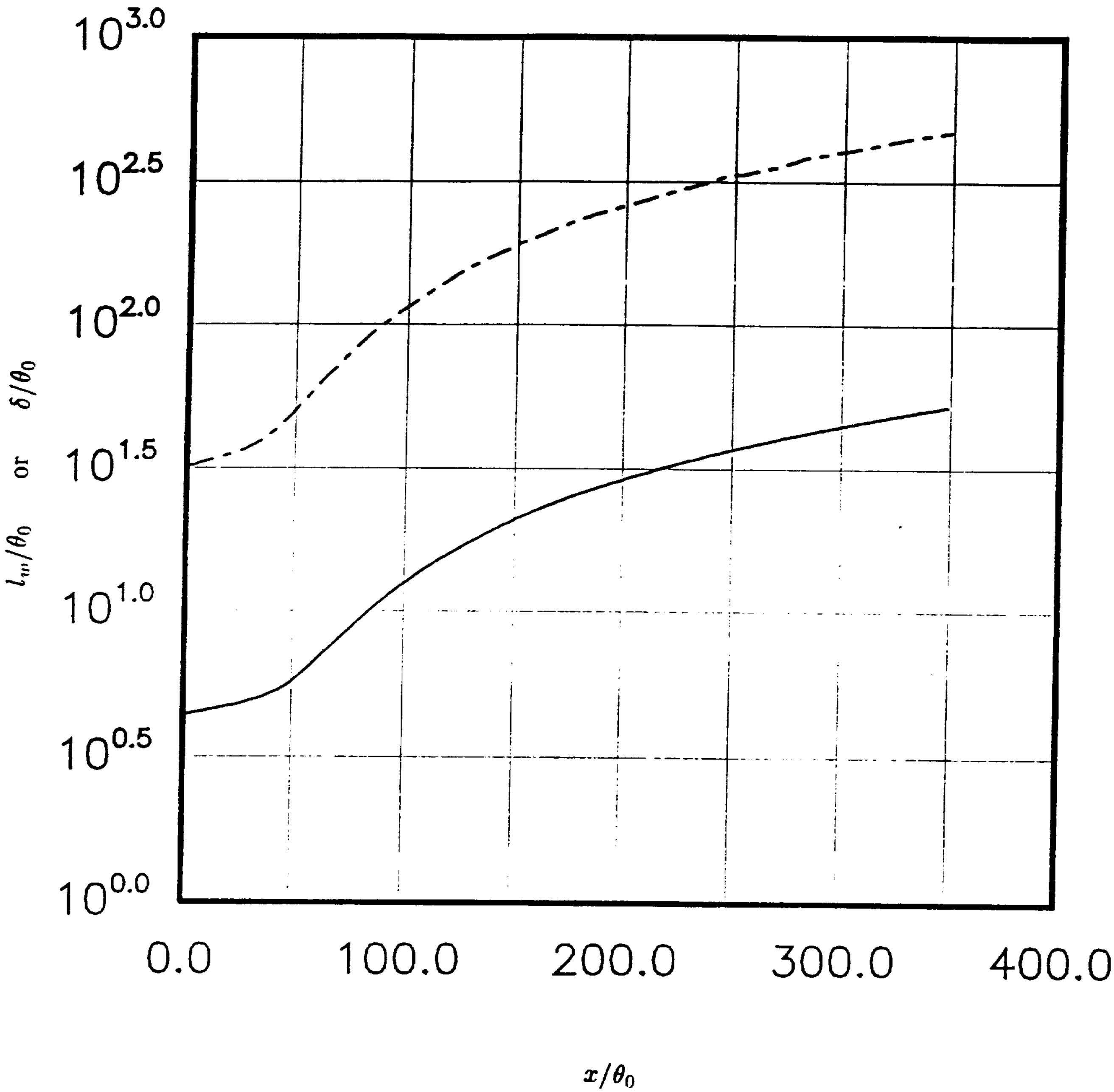

Figure 1. Comparison of length scales. ——, δ; - ——, l_w.

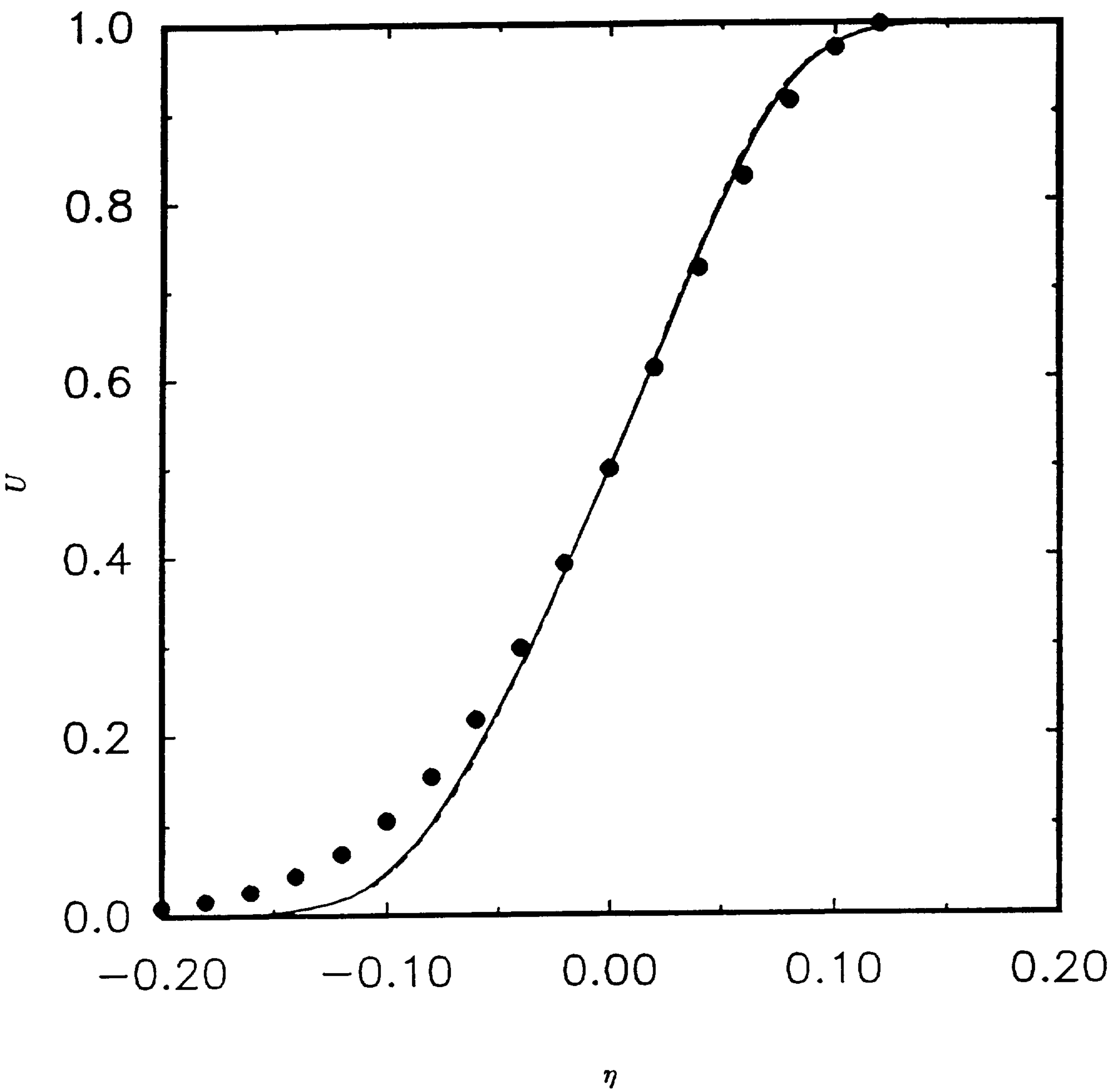

Figure 2. Mean velocity profiles.- - -, $x = 3.82$; - —, 4.43; ——, 5.99; ●, Patel[9].

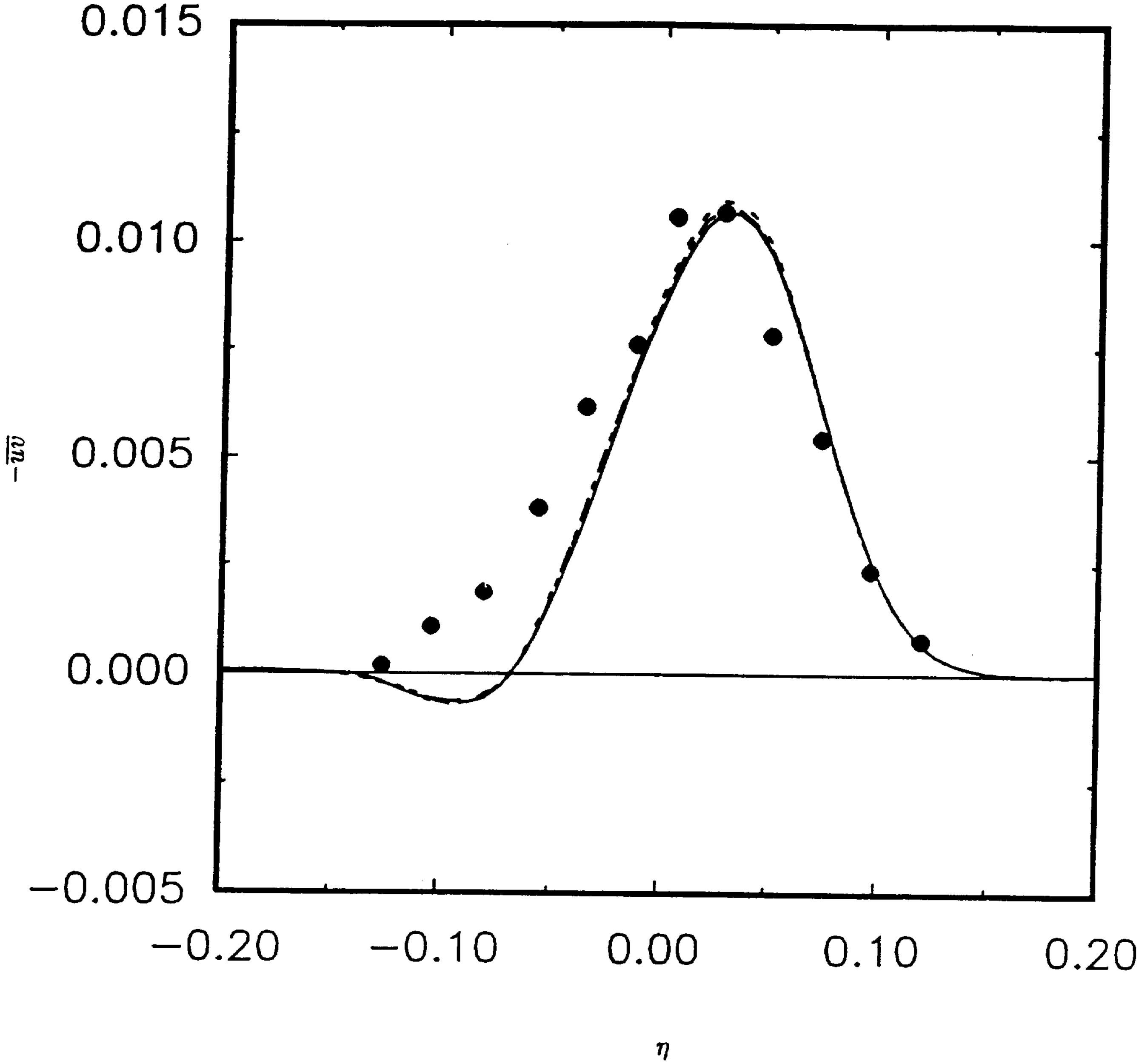

Figure 3. Reynolds shear stress profiles.– – –, $x = 3.82$; - —, 4.43; ——, 5.99; ●, Patel[9].

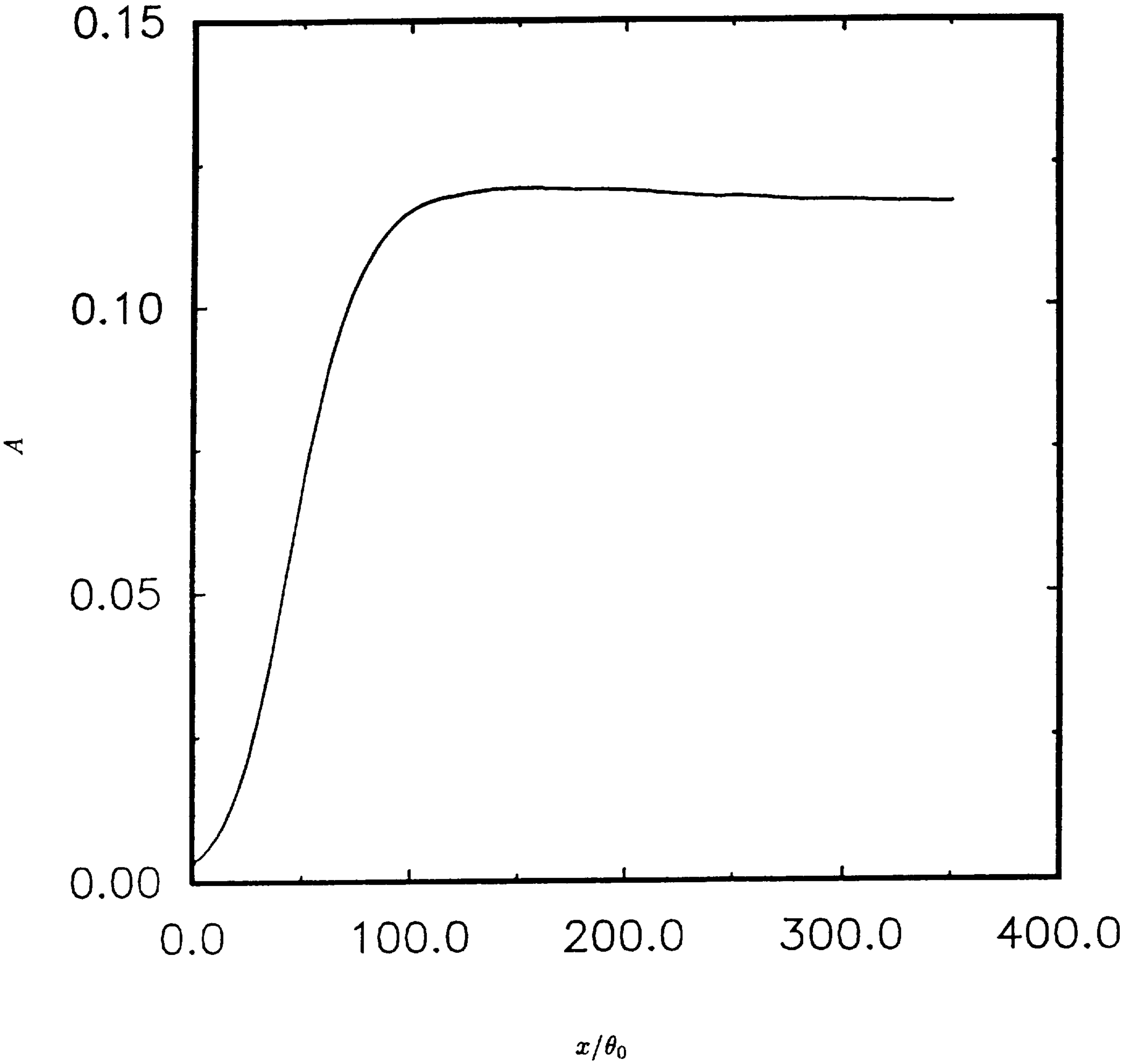

Figure 4. Variation of the wave amplitude with streamwise distance.

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